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HEAT AND MOISTURE TRANSFER IN CONCRETE SLABS

C. L. D. HUANG, H. H. SIANG and C. H. BEST
Kansas State University, Dept. of Mechanical Engineering,
Seaton Hall, Manhattan, KS 66506, U.S.A.*

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Abstract—To predict the moisture distribution in concrete slabs as functions of time, the dependence on the relevant material characteristics must be considered. The phenomena relevant for moisture, pressure, and temperature distribution are coupled. A diffusion theory with a linear or a nonlinear coefficient of diffusivity is not adequate for the description of the drying behavior of hardening concrete. A general mathematical description for the system has to be based upon a non-equilibrium, irreversible flow of heat and moisture. The distribution of pore size may be considered to be the most important parameter affecting moisture transfer in a porous medium. Based upon mass and energy transfer processes and on the liquid–vapor equilibria, a set of governing differential equations is developed for simultaneous heat and mass transfer during the pendular state of drying. Numerical examples, using the theory developed, are illustrated for natural drying of concrete slabs. They show that during the pendular state of drying, both diffusion and evaporation–condensation mechanisms are the governing processes in drying.

NOMENCLATURE

$\mathbf{k}, k$,	permeability tensor, permeability [m^2];
M,	averaged mass for mixture;
M_i,	molecular weight of i-component [kg/mol];
P,	total macroscopic pressure [$kg/m\,s^2$];
K_i,	thermal conductivity of i-component [$kg\,m/s^3\,K$];
K,	effective thermal conductivity tensor [$kg\,m/s^3\,K$];
p_i,	microscopic local pressure of i-component [$kg/m\,s^2$];
p_v,	equilibrium vapor pressure of bulk water [$kg/m\,s^2$];
a,	empirical parameter;
r,	hydraulic porous radius or characteristic length of a porous medium [m];
T,	absolute temperature [K];
e,	emissivity factor;
R,	gas constant [$m\,s^2/kg$];
h,	heat-transfer coefficient.

Greek symbols

θ,	moisture (water) content;
ε,	porosity of the porous system;
$\varepsilon_i(t)$,	volume fraction of the i-component [m^3/m^3];
ϕ,	mole fraction of water vapor of the gaseous component [mol/mol];
ρ_i,	density of the i-component [kg/m^3];
η_i,	shear viscosity of i-component [$kg/m\,s$];
λ,	latent heat of evaporization from the bulk liquid [m^2/s^2];
σ,	surface tension of gas–liquid interface [kg/s^2];
σ_s,	Stefan–Boltzmann [$kg/s^3\,K^4$];
β,	empirical constant [$kg/s^2\,K$];
ζ,	relative permeability $= K_g/K_g^0$;
α,	mass-transfer coefficient [$mol/m^2\,s$].

Subscripts

i,	i-component of the mixture;
s,	of solid;
l,	of liquid;
g,	of gaseous mixture;
gv,	of vapor in a gaseous mixture;
ga,	of air in a gaseous mixture.

I. INTRODUCTION

FOR SEVERAL decades, scientists and engineers have shown considerable interest in the problem of moisture migration in porous media. Chemical engineers, being interested in industrial drying and catalytic operations, have paid considerably more attention to the true sorption phenomena and to the effect of certain characteristics of the surroundings on the responses inside the drying solids, which often exhibit a fine pore structure [1]. Soil scientists have been primarily interested in the movement of moisture at relatively high saturations in the rather coarse pore structures [2]. Three theories have been developed to explain the physical phenomenon of moisture transfer in porous media: the diffusion theory [3], the capillary flow theory (or the nonlinear diffusion theory) [4], and the evaporation–condensation theory [5]. The diffusion theory of moisture transfer was questioned by Ceaglske and Hougen [6] with the statement: "The drying rate of a granular substance is determined not by diffusion but by capillary action". Later, Hougen

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et al. [7] compared the theoretical solutions obtained by the diffusion theory and the capillary flow theory with experimental data for sand, and their results strongly favor the capillary flow theory. However, at the pendular state, the liquid threads in the porous system become progressively discontinuous, and liquid islands form inside the porous system. Motion due to capillary forces is greatly reduced. Under these circumstances, both diffusion and convection in the vapor phase are the primary mechanisms by which the moisture can be transferred. Thus the temperature in the porous system plays the important role in mass transfer, and the temperature gradient becomes a driving force along with the concentration gradient. With the thermal energy equation and Fick's concentration equation, Philip and DeVries [8] and DeVries [9] extended previous treatments of the moisture-transfer problems to include the thermal effects and developed a set of governing equations. Similar governing equations for heat and moisture transfer in porous systems were published also by Luikov [10]. In all of the previous theoretical treatments of moisture migration in porous media, the governing differential equations were inferred in a phenomenological manner. Recently, based on the principles of transport phenomena [11] and nonequilibrium thermodynamics [12], Harmathy [13] derived a set of governing differential equations of moisture migration in porous media, during the pendular state and gave the solution with particular·reference to clay bricks. Huang [14] extended the Harmathy work to the inclusion of the funicular state. Also, Whitaker [15] developed a theory of drying in porous media based on the transport equations with an averaging technique.

Although a large volume of experimental data for drying of concrete slabs has been assembled over many decades of research, a reliable theoretical analysis is not presently available. The structure of the porous space inside a concrete slab is complicated and is strongly influenced by many factors; water–cement ratio, degree of carbonation, and age are just a few [16]. Diffusion theory, which has been used extensively in the past does not give results in agreement with experiments. Capillary flow theory, in which the diffusivity depends on pore water content, is employed by Pihlajavaara [17], Pihlajavaara and Väisänen [18], and Bazant and Najjar [19] to predict the drying rate in concrete slabs. However, the coefficient of diffusivity is a complex function of pore moisture, temperature and other variables of the porous system. It cannot be defined as a simple function of pore moisture as done in [18] and [19]. Therefore, in the present paper, a rigorous analysis will be followed. In other words, the basic equations for mass and heat transfer derived by Huang [14] from the laws of physics will be employed and applied to the study of drying of hydrated cement paste slabs. The properties of cement paste are the primary characteristics of concrete structures. Although the aggregates in cement paste may affect certain properties of concrete, the structure of concrete is mainly determined by the cement paste. Hence, only the hydrated cement pastes will be studied in this paper.

The set of governing differential equations for heat and mass transport in porous media comprises three nonlinear partial differential equations. To find an analytical solution to the set of equations would be extremely difficult. An implicit finite difference method will be employed for solving the equations. The average drying rate and the history of the moisture distribution are obtained for various types of equilibrium sorption relations. Clearly, the equilibrium sorption relation plays the most important role in the drying process.

II. POROUS STRUCTURE OF HYDRATED CEMENT PASTE

The mechanisms of moisture movement in porous media are dominated by the structure of the porous system. The diffusion theory, a single-phase diffusion model, is an oversimplified mechanism to describe the mass transfer in concrete. Although the capillary flow theory is more intricate than the diffusion theory, it is still insufficient to describe such a phenomenon. For the coarser granular soils, the capillary flow theory may yield good results in predicting moisture migration. However, for a porous medium with fine texture, such as paste of concrete, the surface energy of the pore space, which is ignored in both the diffusion and capillary flow theories, significantly affects the movement of moisture [20]. Therefore, a brief description of the structure of cement paste is needed. In past decades, a series of studies concerning the structure and the physical properties of hardened portland cement was published by Powers and Brownyard [16] and Powers [21, 22]. The pore structure of the cement paste can be divided into gel pores and capillary pores. The average diameter of gel pores is 18 Å. The gel pore structure is a result of the growth of very small irregular cryptocrystalline particles, which may have the shapes of fibers, rolled foils, tubes and plane sheets. Therefore, the porous system has an enormous specific surface, which is of the order of $2 \times 10^5\ \mathrm{m^2/kg}$ by dry weight. The capillary pores are scattered in a mass of cement gel. Their diameter is greater than 200 Å [23]. The magnitude of the capillary porosity of cement pastes depends on the water–cement ratio. An increase in water–cement ratio causes an increase in capillary porosity. Structural differences among pastes are primarily the result of the differences in capillary porosity, which in turn depends on the water–cement ratio and the chemical composition of cement. Various sizes of gel pores and capillary pores exist in cement pastes. The pore size distribution is a major characteristic of the porous system. The subject has been discussed by Wittmann [24], and Corey [2] in the area of soil science.

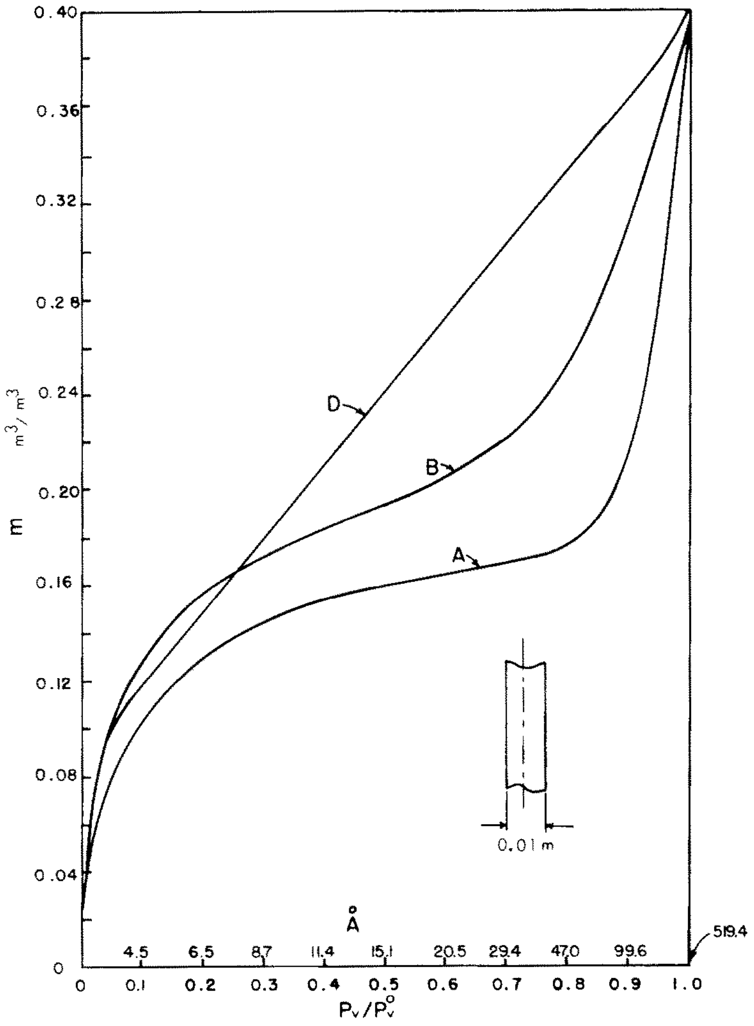

FIG. 1. Generalized liquid–vapor equilibrium relation for a cement paste slab.

III. EQUILIBRIUM SORPTION

A wet porous system is a multi-phase medium. In general it contains all three phases. The liquid and gaseous phases are contained in the porous system with the surface of the solid matrix as their boundaries. In the present paper, the solid matrix is considered, in a macroscopic sense, as a rigid body with homogeneous and isotropic pore structures. The properties of the liquid water phase in the porous system is different, to some extent, from that of the bulk free water. Layers of water molecules next to the surface of the solid matrix are tightly held by van der Waals attraction. The adsorbed water film has a limited mobility and can move by a process of surface diffusion. In the narrow capillary pores, the liquid water beyond the adsorbed liquid film is subject to capillary force and has a higher degree of mobility than the adsorbed liquid film. Convection is the primary transport mechanism for this kind of liquid water. In the gaseous phase, the air and vapor mixture can be transported by both molecular diffusion and molar convection on a macroscopic scale. The water vapor concentration in the mixture can be determined from the equilibrium sorption relation. In turn, the equilibrium sorption relation depends on the characteristics of the porous system, such as the porosity, the specific surface of the solid matrix, and the pore size distribution. In the soil science literature, the moisture in the soil system often is expressed as a function of capillary pressure [2], whereas in chemical science, the moisture content in a porous system is expressed as a function of the relative vapor pressure under a constant temperature (isotherms) [25]. However, for the natural drying processes, the change of temperature in the porous system may be small but not zero. The moisture content thus must be expressed as a function of the relative vapor pressure and temperature. Vassilious and White [26] have proposed that the equilibrium sorption relation could be expressed as a functional relation between the moisture content θ and the "equivalent pore diameter" r.

$$\theta = \theta(r) \quad \text{or} \quad r = r(\theta) \tag{1}$$

where the moisture content can be approximated as

$$\theta = \frac{m}{\varepsilon} \simeq 1 - \frac{\varepsilon_g}{\varepsilon} \qquad (2)$$

by the reason that the water content in the gaseous phase is negligible in comparison with that in the liquid phase. The ε denotes the porosity of the porous system, ε_g the volume fraction of the gaseous phase, and m the volumetric moisture content. This functional relation has been used by Harmathy [13] to study the problem of drying, with particular reference to clay. Actually, the concept of using the functional relation between the moisture content and equivalent pore diameter has been employed by soil physicists in studying the unsaturated flow in soils [2].

From the Kelvin equation, r is expressible as a function of temperature and the relative vapor pressure, as

$$r = -\frac{2\sigma M_w}{\rho_w R} \cdot \frac{1}{T \ln(P_v/P_v^0)} \qquad (3)$$

where σ denotes the surface tension, M_w the molecular weight of water, ρ_w density of water, and R the gas constant.

Thus, a functional relation among moisture content θ, temperature T, and the relative vapor pressure (P_v/P_v^0) can be established. It should be noted that the variable r in (3) should be interpreted as a characteristic length of pore space. Also the analysis assumes that practically all portions of the pore space with pore sizes larger than that given by the Kelvin equation (3) is accessible to the gaseous phase. Experimental evidence for the assumption was given by Corey and Brooks [27]. Figure 1 shows the experimental curves of sorption equilibrium, $\theta = \theta(r)$, for various kinds of the hardened cement paste [16]. Because only the drying processes will be considered in this paper, the curves of the desorption equilibrium are presented. From equations (1)–(3), a relation between the effective porosity and the water content is established as

$$\varepsilon_g = \varepsilon\{1 - \theta[r(\phi, P, T)]\}. \qquad (4)$$

which must be determined experimentally.

IV. BASIC EQUATIONS FOR HEAT AND MASS TRANSFER IN CEMENT PASTE

By assuming the existence of local thermal equilibrium in the porous system, and the gaseous vapor phase as the major factor of mass transfer, one can derive the basic equations for heat and mass transfer from the laws of conservation of mass, momentum, and energy, and the kinetic theory of ideal gases. For one-dimensional mass and heat transfer in a slab of thickness L, the equations are given in the following form [14]:

$$\begin{aligned} A_i\frac{\partial\phi}{\partial t} + B_i\frac{\partial P}{\partial t} + C_i\frac{\partial T}{\partial t} &= D_i\frac{\partial^2\phi}{\partial x^2} + E_i\frac{\partial^2 P}{\partial x^2} + F_i\frac{\partial^2 T}{\partial x^2} + G_i\left(\frac{\partial\phi}{\partial x}\right)^2 \\ &\quad + H_i\left(\frac{\partial P}{\partial x}\right)^2 + I_i\left(\frac{\partial T}{\partial x}\right)^2 + J_i\left(\frac{\partial\phi}{\partial x}\cdot\frac{\partial P}{\partial x}\right) \\ &\quad + K_i\left(\frac{\partial\phi}{\partial x}\cdot\frac{\partial T}{\partial x}\right) + L_i\left(\frac{\partial P}{\partial x}\cdot\frac{\partial T}{\partial x}\right) \end{aligned} \qquad (5)$$

where the coefficients $A_i, \ldots, L_i$ are functions of the dependent variables ϕ, P, T and ε_g. They are defined as follows:

$$A_1 = \bar{Y}(\partial\varepsilon_g/\partial\phi) + \varepsilon_g, \quad A_2 = (1-\phi)(\partial\varepsilon_g/\partial\phi) - \varepsilon_g$$

$$B_1 = \bar{Y}(\partial\varepsilon_g/\partial P) + \varepsilon_g(\phi/P), \quad B_2 = (1-\phi)\bar{O}_p$$

$$C_1 = \bar{Y}(\partial\varepsilon_g/\partial T) - \varepsilon_g(\phi/T), \quad C_2 = -(1-\phi)\bar{O}_T$$

$$D_1 = D\varepsilon_g(M_a/M), \quad D_2 = -D\varepsilon_g(M_w/M)$$

$$E_1 = \zeta\phi(k_g^0/\eta_g), \quad E_2 = \zeta(1-\phi)(k_g^0/\eta_g)$$

$$F_1 = 0, \quad F_2 = 0$$

$$G_1 = -D\bar{O}_\phi(M_a/M), \quad G_2 = D\bar{O}_\phi(M_w/M)$$

$$H_1 = \phi\bar{W}_p(k_g^0/\eta_g), \quad H_2 = (1-\phi)\bar{W}_p(k_g^0/\eta_g)$$

$$I_1 = 0, \quad I_2 = 0$$

$$\begin{aligned} J_1 = D\bar{O}_p(M_a/M) + (k_g^0/\eta_g)[\phi(\partial\zeta/\partial\varepsilon_g) \\ - (\partial\varepsilon_g/\partial\phi) + \zeta] \end{aligned}$$

$$\begin{aligned} J_2 = -D\bar{O}_p(M_w/M) + (k_g^0/\eta_g)[(1-\phi)(\partial\zeta/\partial\varepsilon_g) \\ - (\partial\varepsilon_g/\partial\phi) - \zeta] \end{aligned}$$

$$K_1 = -D\bar{O}_T(M_a/M), \quad K_2 = D\bar{O}_T(M_w/M)$$

$$L_1 = -\phi\bar{W}_T(k_g^0/\eta_g), \quad L_2 = -(1-\phi)\bar{W}_T(k_g^0/\eta_g)$$

$$A_3 = \rho_w Q(\partial\varepsilon_g/\partial\phi), \quad B_3 = \rho_w Q(\partial\varepsilon_g/\partial P) - \varepsilon_g$$

$$\begin{aligned} C_3 &= \rho_w Q(\partial\varepsilon_g/\partial T) \\ &\quad + [(\varepsilon-\varepsilon_g)\rho_w(C_p)_w + \rho_s(1-\varepsilon)(C_p)_s] \\ &\quad + \varepsilon_g P[\phi M_w(C_p)_v + (1-\phi)M_a(C_p)_a]/RT \end{aligned}$$

$$F_3 = K, \quad I_3 = (\partial K/\partial\varepsilon_g)(\partial\varepsilon_g/\partial T)$$

$$\begin{aligned} K_3 &= (\partial K/\partial\varepsilon_g)(\partial\varepsilon_g/\partial\phi) \\ &\quad - DM_aM_w\varepsilon_g P[(C_p)_a - (C_p)_v]/RTM \end{aligned}$$

$$\begin{aligned} L_3 &= (\partial K/\partial\varepsilon_g)(\partial\varepsilon_g/\partial P) \\ &\quad + (K_g^0\zeta P)[M_w(C_p)_v\phi + M_a(C_p)_a(1-\phi)]/RT \end{aligned}$$

in which

$$\begin{aligned} \bar{O}_\phi &= \varepsilon_g[(M_w - M_a)/M] - (\partial\varepsilon_g/\partial\phi) \\ \bar{O}_p &= (\varepsilon_g/P) + (\partial\varepsilon_g/\partial P) \\ \bar{O}_T &= (\varepsilon_g/T) + (\partial\varepsilon_g/\partial T) \\ \bar{Y} &= \phi - (\rho_w RT/M_w P) \\ \bar{W}_p &= (\zeta/P) + (\partial\zeta/\partial\varepsilon_g)(\partial\varepsilon_g/\partial P) \\ \bar{W}_T &= (\zeta/T) - (\partial\zeta/\partial\varepsilon_g)(\partial\varepsilon_g/\partial T). \end{aligned}$$

By examining the three basic equations (5), we note that there are four variables $\phi(x,t)$, $P(x,t)$, $T(x,t)$ and ε_g. Therefore, equations (5) should be incorporated with equation (4) to form a complete set of governing differential equations.

The boundary conditions on the surfaces of the slab are ($i = 0$ for $x = 0$; $i = 1$ for $x = L$)

$$\frac{\partial\phi}{\partial x} = (-1)^i\alpha\left(\frac{RT}{PD}\right)(\phi - \phi_{0i}), \qquad (6a)$$

$$\frac{\partial T}{\partial x} = (-1)^i\left(\frac{h}{K}\right)(T - T_{0i}) \qquad (6b)$$

and

$$P = P_{0i}. \tag{6c}$$

where α denotes the mass-transfer coefficient, D the diffusion coefficient, h the heat-transfer coefficient, and k thermal conductivity. ϕ_0, T_0, and P_0 respectively are the mole fraction of water vapor, temperature, and pressure of the surroundings of the porous system. For a slab surrounded by air at normal conditions (the air pressure is at atmospheric pressure and the temperature is between 0°C and 60°C), the transfer coefficients may be expressed as [13] ($i = 0$ for $x = 0$; $i = 1$ for $x = L$),

$$\alpha = \alpha_F + 1.63 \times 10^{-5}[(T_{oi} - T)/B]^{1/4} \tag{7}$$

$$h = 2.75 \times 10^8 \alpha + \sigma_s e(T_{ei}^4 - T^4)/(T_{ei} - T) \tag{8}$$

$$K = [K_g^n \varepsilon_g + K_l^n \varepsilon_l + K_s^n (1 - \varepsilon)]^{1/n} \tag{9}$$

where α_F denotes the mass-transfer coefficient due to forced convection. B is a characteristic length of the slab, σ_s the Stefan–Boltzmann constant, T_e the temperature of the enclosure of the system and n the topological constant of the porous system. K_g, K_l and K_s are the conductivities for gas, liquid, and solid matrix respectively; ε, ε_g and ε_l are the porosity of the porous system, the volume fraction for gas, and the volume fraction for liquid respectively.

The initial conditions for the present study are given as follows:

$$\phi(x,0) = \phi_0,\; P(x,0) = P_{\text{atm}},\; T(x,0) = T_0. \tag{10}$$

Therefore equations (4)–(6) and (10) form a nonlinear boundary value problem for the simultaneous mass and heat transfer in a cement paste slab.

V. NUMERICAL ANALYSIS AND CALCULATION

Obtaining an analytical solution for the nonlinear partial differential equations governing flows through porous media equations (4, 6, 10) would not be possible. In this paper, an implicit finite difference scheme is employed for obtaining numerical results. The first step is to replace the equations by a set of algebraic finite backward-in-time equations, which give the relationships among the dependent variables ϕ, P, and T at neighboring points in an (x, t) space. The numerical solution of the simultaneous algebraic equations thus obtained yields the values of the dependent variables at the pre-assigned grid points throughout the domain investigated. In this paper, one-dimensional heat and mass flows are considered.

In the set of algebraic equations, there are one space increment Δx and one temporal increment Δt. The finer meshes for Δx and Δt will result in a smaller error. Yet, the magnitude of Δx and Δt cannot be chosen arbitrarily. It can be shown [28] that in order to have a stable solution of equations (4) and (5), one must properly choose Δx and Δt such that:

$$\Delta t/(\Delta x)^2 \leqslant \tfrac{1}{2}C$$

where C is a parameter determined by the ratios of the coefficients $A_i, \ldots, K_i$. For the problem considered, the parameter C varies with time t because those coefficients also vary with time. In order to achieve a stable computation, a considerably smaller value of Δt must be used at the beginning stage of the computational simulation than at the advanced stages, due to errors associated with the initial guess of sorption equilibrium.

The derivatives of ϕ with respect to x at a given time t can be expressed as:

$$\left[\frac{\partial\phi(t)}{\partial x}\right]_i \simeq \frac{\phi_{i+1}(t-\Delta t) - \phi_{i-1}(t-\Delta t)}{2\Delta x} + \frac{\Delta\phi_{i+1} - \Delta\phi_{i-1}}{2\Delta x} \tag{11}$$

$$\left[\frac{\partial\phi(t)}{\partial x}\right]_i^2 \simeq \left[\frac{\phi_{i+1}(t-\Delta t) - \phi_{i-1}(t-\Delta t)}{2\Delta x}\right]^2 + 2\cdot\frac{\phi_{i+1}(t-\Delta t) - \phi_{i-1}(t-\Delta t)}{2\Delta x} \times \frac{\Delta\phi_{i+1} - \Delta\phi_{i-1}}{2\Delta x} \tag{12}$$

$$\left[\frac{\partial^2\phi(t)}{\partial x^2}\right] \simeq \frac{\phi_{i+1}(t-\Delta t) - 2\phi_i(t-\Delta t) + \phi_{i-1}(t-\Delta t)}{(\Delta x)^2} + \frac{\Delta\phi_{i+1} - 2\Delta\phi_i + \Delta\phi_{i-1}}{(\Delta_x 2)} \tag{13}$$

and

$$\left[\frac{\partial\phi(t)}{\partial t}\right]_i \simeq \frac{\Delta\phi_i}{\Delta t} \tag{14}$$

where $\Delta\phi_i = \phi_i(t) - \phi_i(t - \Delta t)$.

Similar finite difference expressions can be written for the derivatives of the other dependent variables P and T. For example, the cross-product of derivatives is

$$\begin{aligned}\left[\frac{\partial\phi(t)}{\partial x}\right]_i \left[\frac{\partial P(t)}{\partial x}\right]_i &\simeq \frac{\phi_{i+1}(t-\Delta t) - \phi_{i-1}(t-\Delta t)}{2\Delta x} \times \frac{P_{i+1}(t-\Delta t) - P_{i-1}(t-\Delta t)}{2\Delta x} \\ &+ \frac{\phi_{i+1}(t-\Delta t) - \phi_{i-1}(t-\Delta t)}{2\Delta x} \cdot \frac{\Delta P_{i+1} - \Delta P_{i-1}}{2\Delta x} \\ &+ \frac{P_{i+1}(t-\Delta t) - P_{i-1}(t-\Delta t)}{2\Delta x} \cdot \frac{\Delta\phi_{i+1} - \Delta\phi_{i-1}}{2\Delta x}.\end{aligned} \tag{15}$$

Similar finite-difference expansions can be written for the other cross-product of derivatives of other dependent variables.

It should be noted that the coefficients $A_i, \ldots, K_i$ ($i = 1,2,3$) are functions of the three dependent

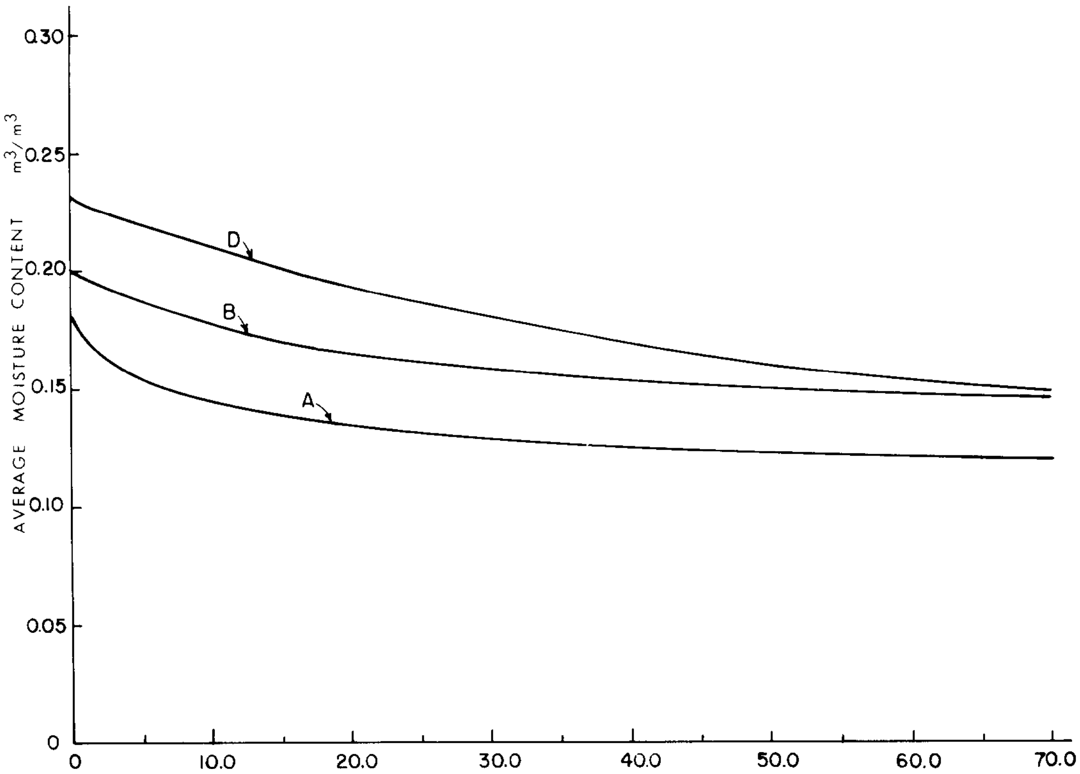

FIG. 2. Average moisture drying rate.

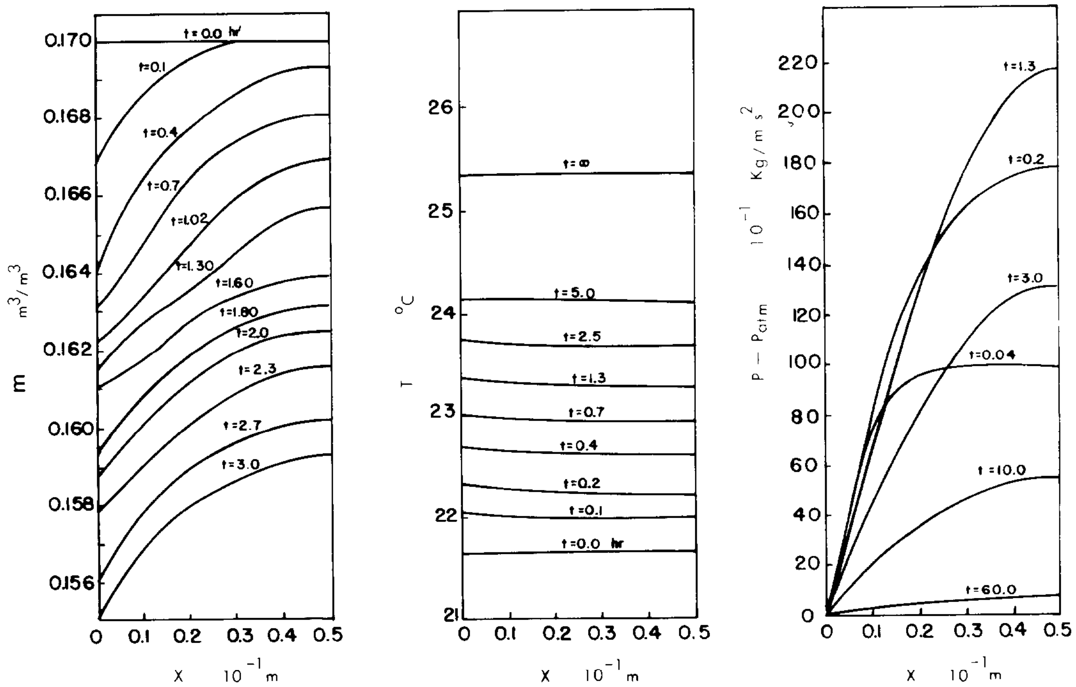

FIG. 3. Histories of moisture, temperature and pressure distribution for earlier drying state in a 0.01 m thick cement paste slab (Case A).

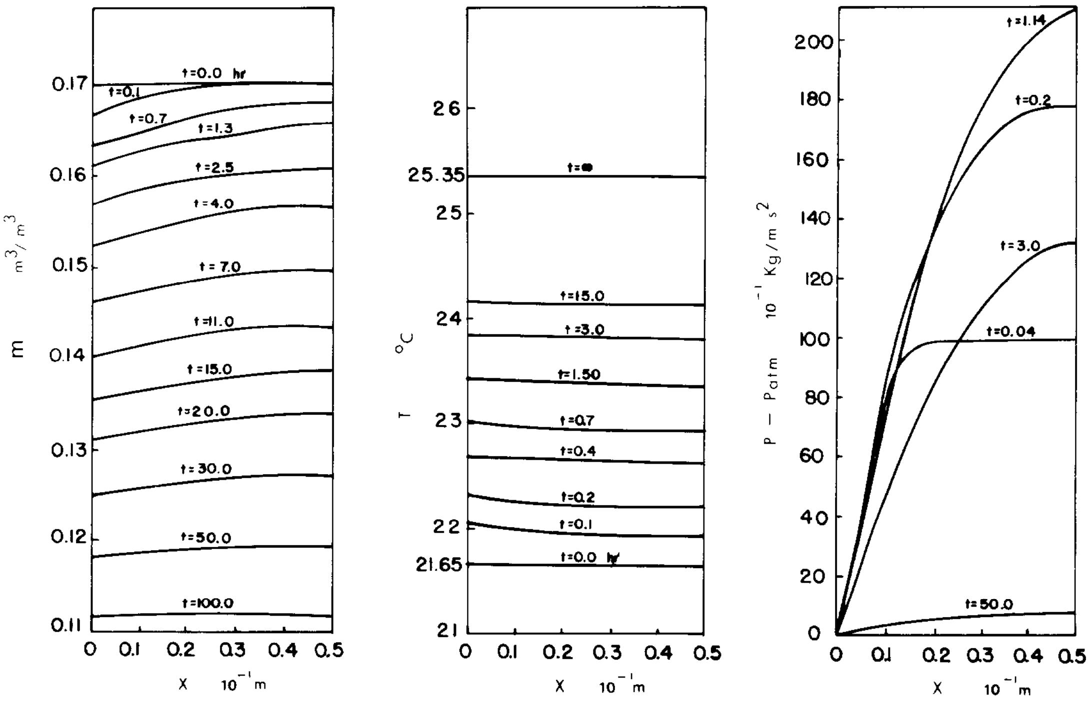

FIG. 4. Distributions of moisture, temperature and pressure in a 0.01 m thick cement paste slab (Case A).

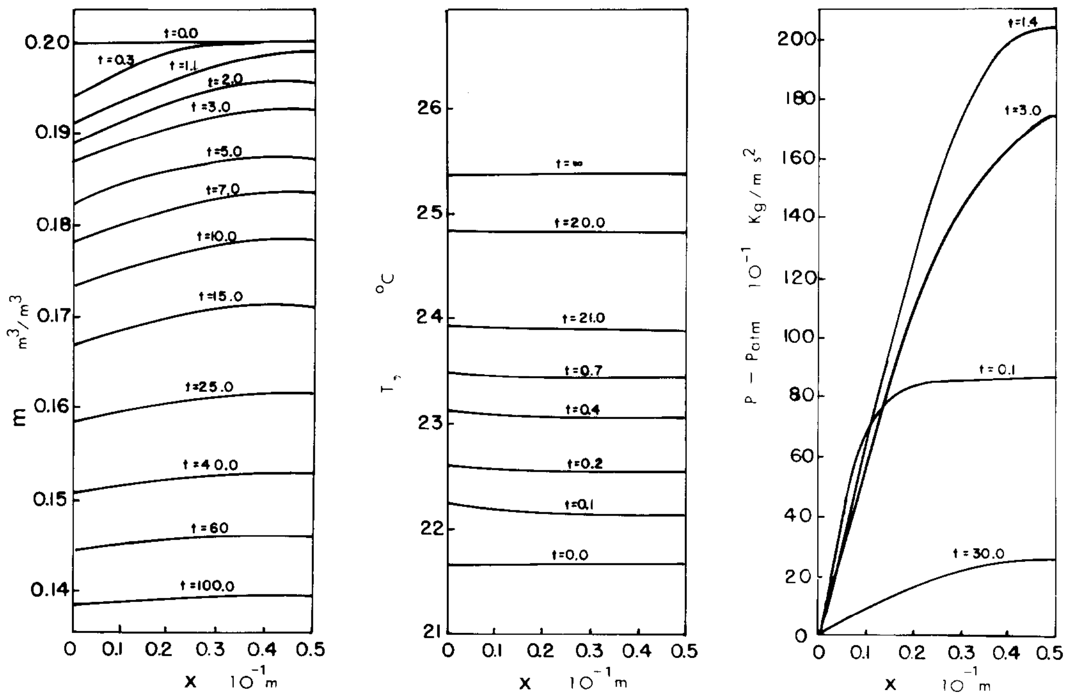

FIG. 5. Distributions of moisture and pressure in a 0.01 m thick cement paste slab (Case B).

variables and the following material characteristics: specific heat, density and thermal conductivity of all components of the porous system, porosity and permeability of the solid matrix, diffusion coefficient of water vapor in air, viscosity of the gaseous mixture, sorption equilibrium relation and heat of sorption. The numerical solutions for the histories of moisture distribution, temperature and pressure in a concrete slab are obtained with a digital computer. Also, the average drying rate for a cement paste slab is calculated.

Three typical liquid–vapor equilibrium curves for cement paste are shown in Fig. 1. Curve A shows that the pore sizes of the cement paste are quite uniform and rather coarse; Curve D shows that the pore sizes of the cement paste are well distributed

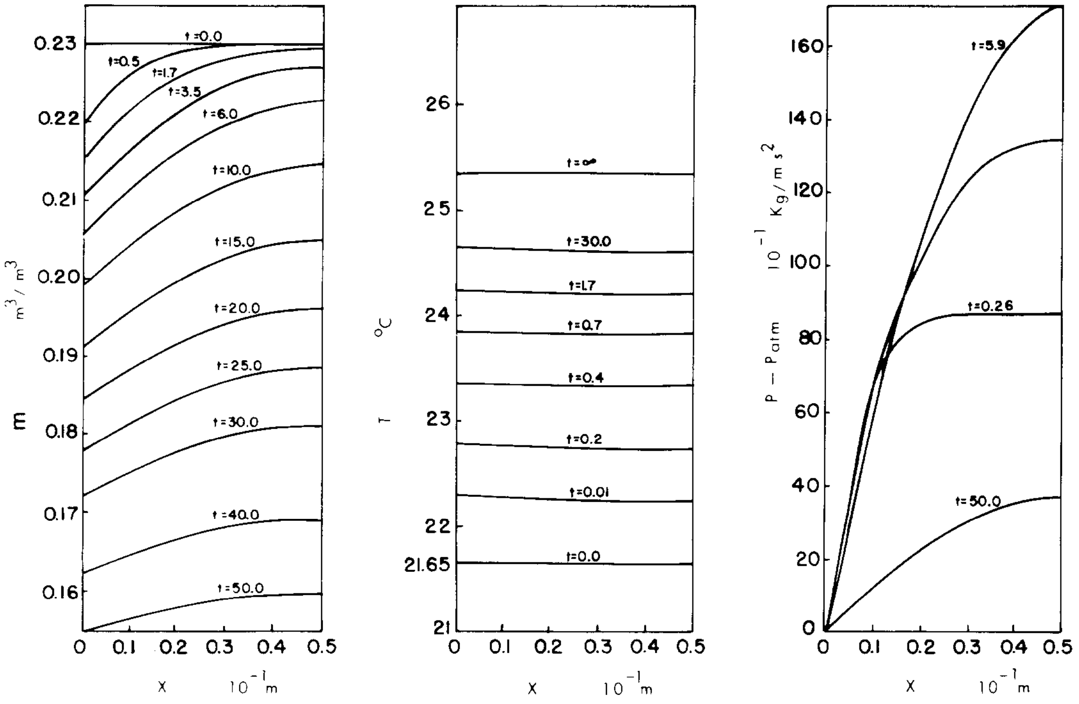

FIG. 6. Distributions of moisture, temperature and pressure in a 0.01 m thick cement paste slab (Case D).

and that the structure of the solid matrix has a fine texture. In other words, cement paste D has a higher value of specific surface than cement paste A. The curve B shows another pore structure of cement paste.

Figure 2 shows the average drying rate for cement pastes A, B, and D. Evidently, cement paste A will dry faster than the other cases. Figures 3 and 4 show the histories of moisture, temperature, and pressure distribution inside a slab of cement paste A with a thickness of 1.0 cm. Some irregularities appear in the moisture distribution curves. The irregularities are caused by the capillary actions of moisture in intensities of the solid matrix [6]. Figures 5 and 6 give the histories of moisture, temperature, and pressure distribution in the slabs of cement paste B and D, respectively.

VI. CONCLUSIONS AND REMARKS

From the history of moisture distribution, the curves reveal that at high pore saturation (funicular saturation), the moisture movement is relatively independent of the properties of the porous system and dependent on the parameters characterizing the surroundings, such as temperature, velocity, and the relative humidity of the ambient air. When the liquid threads in the porous medium begin to break down, and capillary action in the pore spaces set in, the internal characteristics of the porous system play the primary role in moisture migration. The curve of moisture distribution is no longer convex. At the pendular saturation stage, liquid moisture has to be vaporized first in order to move from one location to another [29]. The drying rate varies with time and is different from one location to another throughout the thickness of the slab. In general, the drying rate is slowed down. At low moisture saturation, vapor flux is the only mechanism of moisture transfer in the system. Because of the phenomenon of pure adsorption, the drying rate becomes very, very slow.

Evidently, the sorption equilibrium curve is one of the most important factors in determining the mechanisms of moisture transfer. In turn, it determines the drying rate in the desorption processes. The rate of moisture transfer is rather high in the solid matrix with a fairly uniform pore structure, a porous medium composed mostly of particles of a particular size. The uniform porous medium has a low value of specific surface. However, good cement pastes have a high specific surface, and thus can hold a significant amount of moisture in the equilibrium state. Therefore, moisture migration in a good cement paste is much slower than that in a poor cement paste with a high magnitude of capillary porosity, or in a granular material such as sand.

In the present theory, the solid matrix is assumed to be a rigid structure, i.e. the porosity of the solid matrix is constant. Therefore, the problems of creep, hydration, and carbonation in concrete are beyond the scope of this study.

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APPENDIX

Table 1. Values of the constants used in the equations (cement paste)

Type of const.	Symbol	Value	Unit
Physic. const.	R	8.3149×10^{-14}	kg m^2/s^2 K mol
	σ_s	5.670×10^{-8}	kg/s^3 K^4
	$(C_p)_a$	1.0063×10^3	m^2/s^2 K^2
	$(C_p)_s$	0.879×10^3	m^2/s^2 K^2
	$(C_p)_v$	1.8646×10^3	m^2/s^2 K^2
	$(C_p)_w$	4.1793×10^3	m^2/s^2 K^2
	D	0.256×10^{-4}	m^2/s
Physical properties	e	0.8	
	k_g	0.02613	kg m/s^3 K
	k_s	1.4422	kg m/s^2 K
	k_w	0.616	kg m/s^3 K
	M_a	28.952×10^{-3}	kg/mol
	M_w	18.016×10^{-3}	kg/mol
	ε	0.43	m^3/m^3
	η_g	1.83×10^{-5}	kg/m s
	K_g^0	2.50×10^{-14}	m^2
	λ	2.4418×10^6	m^2/s^2
	ρ_s	2.2×10^3	kg/m^3
	ρ_w	0.99707×10^3	kg/m^3
Geom. const.	L	0.01	m
	B	0.1839	m
Empirical const.	a	1.209658×10^{-11}	m s^2/kg
	b	5080	K
	n	0.25	
	β	0.167×10^{-3}	kg/s^2 K
	v	1.0	
	σ_0	121.2×10^{-3}	kg/s^2
Initial conditions	P_{atm}	1.01325×10^5	kg/m s^2
	$(m_{ini})_A$	0.17	m^3/m^3
	$(m_{ini})_B$	0.20	m^3/m^3
	$(m_{ini})_D$	0.23	m^3/m^3
	T_{ini}	294.8	K
Boundary conditions	ϕ_0	$\phi_{00} = 0.003548$	mol/mol
		$\phi_{01} = 0.003548$	mol/mol
	T_0	$T_{00} = 298.5$	K
		$T_{01} = 298.5$	K
	T_e	$T_{e0} = 298.5$	K
		$T_{e1} = 298.5$	K
	α_F	0.0	mol/m^2 s

TRANSFERT DE CHALEUR ET D'HUMIDITE DANS DES PLAQUES DE BETON

Résumé—Pour prédire la distribution d'humidité dans des plaques de béton en fonction du temps, on considère les caractéristiques du matériau et leur dépendance. Les phénomènes concernant les distributions d'humidité, de pression et de température sont couplés. Une théorie de diffusion, avec un coefficient de diffusion linéaire ou non, n'est pas adaptée à la description du séchage du béton. Un modèle mathématique général est basé sur l'écoulement irréversible et hors d'équilibre de la chaleur et de l'humidité. La distribution de la taille des pores peut être considérée comme le paramètre le plus important qui intervient dans le transfert d'humidité dans un milieu poreux. Basé sur les mécanismes de transfert de chaleur et de masse et sur l'équilibre liquide–vapeur, un système d'équations aux dérivées partielles est formulé pour les transferts simultanés de chaleur et de masse pendant l'état pendulaire de séchage. Des exemples numériques, à partir de la théorie développée, sont illustrés pour le séchage naturel de plaques de béton. Ils montrent que pendant l'état pendulaire de séchage, à la fois la diffusion et l'évaporation–condensation sont les mécanismes qui gouvernent le séchage.

WÄRMEÜBERGANG UND FEUCHTIGKEITSTRANSPORT IN BETONPLATTEN

Zusammenfassung—Bei der Berechnung der Feuchtigkeitsverteilung in Betonplatten als Funktion der Zeit muß die Abhängigkeit des Vorgangs von den wesentlichen Stoffgrößen betrachtet werden. Die Phänomene, die Feuchtigkeits-, Druck- und temperaturverteilung bestimmen, sind gekoppelt. Eine Diffusionstheorie mit einem linearen oder nichtlinearen Diffusionskoeffizienten ist zur Beschreibung des Trockenvorgangs von abbindendem Beton nicht ausreichend. Bei einer allgemeinen mathematischen Beschreibung des Systems muß von einer irreversiblen Nicht-Gleichgewichtsströmung von Wärme und Feuchtigkeit ausgegangen werden. Die Verteilung der Porengröße kann als der Parameter mit dem größten Einfluß auf den Feuchtigkeitstransport in einem porösen Medium angesehen werden. Ausgehend von Stoff—und Energieübertragungsprozessen und dem Flüssigkeits-Dampf-Gleichgewicht wurde ein Satz von maßgeblichen Differentialgleichungen für gleichzeitigen Wärmeübergang und Stoffaustausch beim Trocknen entwickelt. Mit Hilfe der entwickelten Theorie werden Zahlenbeispiele für natürliches Trocknen von Betonplatten erläutert. Die Beispiele zeigen, daß sowohl Diffusion als auch Verdampfungs-Kondensationsmechanismen den Vorgang bestimmende Phänomene beim Trocknen sind.

ТЕПЛО- И ВЛАГООБМЕН В БЕТОННЫХ ПЛИТАХ

Аннотация — Определяется распределение влаги в бетонных плитах с соответствующими характеристиками материала. Проводится совместное рассмотрение явлений, связанных с распределением влаги, давления и температуры. Уравнение диффузии с линейным или нелинейным коэффициентом диффузии не может использоваться для описания процесса сушки твердеющего бетона. Общее математическое описание системы должно основываться на неравновесных необратимых процессах переноса тепла и влаги. Распределение пор по размерам является, вероятно, наиболее важным параметром, влияющим на перенос влаги в пористой среде. С учётом процессов переноса массы и энергии, а также равновесия между жидкостью и паром выведена система основных дифференциальных уравнений для совместного переноса тепла и массы в неустановившемся режиме сушки. Предложенный метод использован для численного расчёта процесса естественной сушки бетонных плит. Результаты показывают, что в неустановившемся режиме сушки определяющими процессами являются как диффузионный, так и испарительноконденсационный механизмы.